PSU-PKU
International Undergraduate Mathematics Summer School
2014

“Geometry and Mathematical Physics”

Course Description

The three-week school will be accessible to students familiar with Math 220, Math 230, Math 250 and Math 312. Any additional background that may be required from Math 497 will be covered as needed.

Differential geometry can be described as the application of the tools of calculus to questions of geometry. In the nineteenth century, Gauss displayed the extent to which calculus (in particular, the first two derivatives) determines basic properties of curves and surfaces, at least locally. These results can properly be called the beginning of “classical” differential geometry. In particular, the first notions of the tangent space would allow the techniques of linearization and ultimately the tools of linear algebra to be brought to bear on geometric questions. In connection to other fields of Mathematics, Sophus Lie, under the influence of Felix Klein, introduced the notion of transformation groups in the course of investigating differential equations. This work led to the notion of a vector field and its integral curves which were applied to physical questions such as energy conservation laws, Hamilton's equations etc. Later, into the early twentieth century through the continued work of F. Engel and E. Cartan, these notions led to what are today known as *symplectic* and *contact* structures on manifolds. It is impossible to overstate the importance of the notion of the tangent space for the field of differential geometry. In fact, today it is possible to describe differential geometry as “the study of smoothly varying structures on the tangent bundle.” The school aims to present the core notions of the field and elaborate on the corresponding techniques.

Course Syllabus

1. **Introduction** – Vector spaces and Linear Transformations, Dual of Vector Space, Forms, Pullbacks, Inner Products, Symplectic Linear Forms.
2. **Advanced Calculus** – Tangent spaces, Linear Maps between Tangent Spaces, Derivatives, Diffeomorphisms, ODEs, Vector Fields and Integral Curves.
3. **Differential Forms and Tensors –** Alternating Linear Forms, Algebra of Differential Forms, Integration, The Lie Derivative, Tensors (optional).
4. **Symplectic Geometry** – Hamiltonian Mechanics and Phase Space, Basic Consepts, Symplectic Diffeomorphisms (optional), Symplectic and Hamiltonian Vector Fields, Geometric Sets in Symplectic Spaces, Hypersurfaces and Symplectic Invariants (optional).